Planetary Diameters in the *Sūrya-siddhānta*

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Abstract. This paper discusses a rule given in the Indian astronomical text *Sūrya-siddhānta* for computing the angular diameters of the planets. I show that this text indicates a simple formula by which the true diameters of these planets can be computed from their stated angular diameters. When these computations are carried out, they give values for the planetary diameters that agree surprisingly well with modern astronomical data. I discuss several possible explanations for this, and I suggest that the angular diameter rule in the *Sūrya-siddhānta* may be based on advanced astronomical knowledge that was developed in ancient times but has now been largely forgotten.

In chapter 7 of the *Sūrya-siddhānta*, the 13th *śloka* gives the following rule for calculating the apparent diameters of the planets Mars, Saturn, Mercury, Jupiter, and Venus:

> 7.13. The diameters upon the moon's orbit of Mars, Saturn, Mercury, and Jupiter, are declared to be thirty, increased successively by half the half; that of Venus is sixty.[1]

The meaning is as follows: The diameters are measured in a unit of distance called the *yojana*, which in the *Sūrya-siddhānta* is about five miles. The phrase "upon the moon's orbit" means that the planets look from our vantage point as though they were globes of the indicated diameters situated at the distance of the moon. (Our vantage point is ideally the center of the earth.) Half the half of 30 is 7.5. Thus the *śloka* says that the diameters "upon the moon's orbit" of the indicated planets are given by 30, 37.5, 45, 52.5, and 60 *yojanas*, respectively.

The next *śloka* uses this information to compute the angular diameters of the planets. The verse allows for the variable distance of the planets from the earth, but for the purposes of this paper it is enough to consider the angular diameters at mean planetary distances. The diameters upon the moon's orbit were given in *śloka* 7.13 for the planets at these mean distances from the earth. The *Sūrya-siddhānta* says that there are 15 *yojanas* per minute of arc at the distance of the moon. Thus the mean angular diameters of the planets can be computed by dividing the diameters upon the moon's orbit by 15. Table 1 lists the results of this computation and compares them with other estimates of planetary angular diameters.

The *Sūrya-siddhānta* figures are roughly the same size as the planetary angular diameters reported by the 2nd century Alexandrian astronomer Claudius Ptolemy in his book *Planetary Hypotheses*.[2] Ptolemy attributed his angular diameters to the Greek astronomer Hipparchus,

Planet	*Sūrya -sidd.*	Ptolemy	Tycho Brahe	Modern min.	Modern max.
Mars	2.000	1.567	1.667	.067	.450
Saturn	2.500	1.741	1.833	.250	.350
Mercury	3.000	2.089	2.167	.067	.200
Jupiter	3.500	2.611	2.750	.333	.817
Venus	4.000	3.133	3.250	.150	1.233

Table 1. Angular diameters of planets in minutes of arc. The modern angular diameters are for the greatest and least distances of the planets from the earth.

but he did not say how they were measured. According to the historian of astronomy Noel Swerdlow, no earlier reports of planetary angular diameters are known, and Ptolemy's angular diameters were reproduced without change by later Greco-Roman, Islamic, and European astronomers up until the rise of modern astronomy in the days of Galileo, Kepler, and Tycho Brahe.[3]

Brahe's figures were obtained by sighting through calibrated pinholes by the naked eye. They are very similar to Ptolemy's, and they are clearly much larger than the angular diameters measured in more recent times by means of telescopes.[4] It is well known that a small, distant light source looks larger to the naked eye than it really is. This phenomenon makes it likely that angular diameters of planets would inevitably have been over-estimated by astronomers before the age of the telescope.

It might appear that the angular diameters given in the *Sūrya-siddhānta* are also derived from naked eye measurements. However, there is another way to interpret them. This is based on the sizes of the planetary orbits, as given in the *Sūrya-siddhānta*.

Ślokas 12.85–90 of the *Sūrya-siddhānta* give the circumferences in *yojanas* of the planetary orbits, and these circumferences are listed in Table 2. The orbits are represented as simple circles centered on the earth, and they represent the mean motions of the planets. For Mercury and Venus, the mean planetary position is the same as the position of the sun (since these planets seem to orbit the sun as the sun orbits the earth). Thus the orbital circumferences in the table are the same for Mercury, Venus, and the sun. For Mars, Jupiter, and Saturn, the mean position corresponds to the average motion of the planet in its heliocentric orbit. This averages out the prograde and retrograde motions corresponding to the earth's motion around the sun.

Given these orbits, we can calculate the true diameters that the planets must have in order to appear to have the diameters "upon the moon's orbit" given in *śloka* 7.13. The formula is simply,

$$\mathrm{D(planet)} = \mathrm{DM(planet)} \times \mathrm{ORB(planet)} / \mathrm{ORB(moon)} \qquad (1)$$

where D(planet) is the true diameter of the planet as deduced from *Sūrya-siddhānta*, DM(planet) is the planet's diameter upon the moon's orbit, ORB(planet) is the circumference of the planet's orbit from Table 2, and ORB(moon) is the circumference of the moon's orbit.

The *Sūrya-siddhānta* also gives explicit values for three other planetary diameters. *Ślokas* 1.59 and 4.1 give the diameters of the earth, sun, and moon as 1,600, 6,500, and 480 *yojanas* respectively. Indian astronomers such as Parameśvara (A.D. 1380–1450) were acquainted with

Planet	Modern period (days)	*Sūrya-siddhānta* orbit (*yojanas*)
Moon	27.32166	324,000
Mercury	365.257	4,331,500
Venus	365.257	4,331,500
Sun	365.257	4,331,500
Mars	686.980	8,146,909
Jupiter	4,332.587	51,375,764
Saturn	10,759.202	127,668,255

Table 2. Geocentric planetary periods and orbital circumferences. The planetary periods are as given in modern astronomy.[5] The orbital circumferences are given in the *Sūrya-siddhānta*, texts 12.85–90.[6]

the method of measuring the earth's circumference by astronomically determining the latitudes of two points separated by a known distance on a north-south line.[7] Thus it seems reasonable to suppose that the *Sūrya-siddhānta's* figure for the size of the earth might be realistic. This enables us to calculate the number of miles per *yojana* by dividing the modern diameter of the earth in miles by 1,600. If we use 8,000 miles as the approximate diameter of the earth, we get 5 miles per *yojana* by this calculation.[8]

Using our figure of 5 miles/*yojana*, we can convert our other planetary diameters into miles and compare them with modern values. This is done in Table 3.

A noteworthy feature of these calculated diameters is that their errors relative to modern values are quite small for four planets: the moon, Mercury, Mars, and Saturn. The calculated diameters are too small by about 50% for Venus and Jupiter, and the sun's calculated diameter is small by 96%.

It is easy to see why the diameter for the moon should be reasonably accurate. The dimensions of the moon and its orbit were well known in ancient times, and we can see that the lunar diameter given by Ptolemy in his *Planetary Hypotheses* falls within about 7% of the modern value. It is therefore not surprising to see a lunar diameter that falls within 11% of the modern value in an Indian astronomical treatise.[11] It is also easy to see why the diameter for the sun is far too small. Ancient astronomers tended to greatly underestimate the earth-sun distance, and Table 2 shows that this also happened in the *Sūrya-siddhānta*. The angular diameter of the sun is easily seen to be about the same as that of the moon—about 1/2 degree. This angular diameter, combined with a small earth-sun distance, leads inevitably to a small estimate for the diameter of the sun.[12] Note that Ptolemy's solar diameter figure is similar to the *Sūrya-siddhānta's*.

So the diameters given for the sun and the moon can be readily understood. But what about the other planets? Their computed angular diameters are too large, but presumably no one could have known this before the days of telescopic observation. Their orbital circumferences are also too small, as is generally the case in ancient astronomical systems. But how is it that these two errors cancel out for three planets out of five to yield diameters within 10% of modern values?

Perhaps this happened simply by chance. This possibility can be evaluated by proposing a model in which diameters are chosen at random. One can then check to see if the observed

Planet	Modern	*Sūrya -sidd.*	% Error	Ptolemy	% Error
Moon	2,160.00	2,400.00	11.11	2,312.33	7.05
Mercury	3,100.00	3,007.99	-2.97	293.63	-90.53
Venus	7,560.00	4,010.65	-46.95	2,246.27	-70.29
Sun	865,110.00	32,500.00	-96.24	43,604.00	-94.96
Mars	4,191.00	3,771.72	-10.00	9,060.57	116.19
Jupiter	86,850.00	41,623.88	-52.07	34,552.86	-60.22
Saturn	72,000.00	73,882.09	2.61	34,090.40	-52.65

Table 3. Planetary diameters in miles. The *Sūrya-siddhānta* diameters were taken from *śloka* 4.1 for the sun and moon, and they were computed by formula (1) for the other planets. These diameters were converted to miles by multiplying by 5 miles/*yojana*.[9] The Ptolemaic diameters were given in earth diameters in Ptolemy's *Planetary Hypotheses* and converted into miles by multiplying by the modern diameter of the earth.[10] The error percentages compare the *Sūrya-siddhānta* and Ptolemaic diameters with the corresponding modern diameters.

correlation between modern and *Sūrya-siddhānta* diameters is likely to show up in this model. The problem with doing this is that it is difficult to propose a realistic probabilistic model of how ancient people would have generated astronomical data. However, it is possible to set up a simple model in which it is assumed that all planetary diameters, ancient and modern, are given by positive random numbers. It is easy to show that the observed correlation between modern and *Sūrya-siddhānta* diameters is highly unlikely to arise by chance, given this model. This is discussed in the appendix.

If the observed correlation did not happen by chance, then perhaps it happened by design. One possible hypothesis is that at some time in the past, ancient astronomers possessed realistic values for the diameters of the planets. One can suppose that they might have acquired this knowledge during a forgotten period in which astronomy reached a high level of sophistication and planets were observed using telescopes or other advanced instruments. Later on, much of this knowledge was lost, but fragmentary remnants were preserved and eventually incorporated into texts such as the *Sūrya-siddhānta*. In particular, the real diameters of the planets were later combined with erroneous orbital circumferences to compute the diameters "upon the moon" given in *śloka* 7.13. These figures were then accepted because they gave realistic values for the angular diameters of the planets as seen by the naked eye.

This hypothesis is supported by the fact that the *Sūrya-siddhānta* diameters of Jupiter and Venus in Table 3 are almost exactly half of the corresponding modern diameters. If we multiply these *Sūrya-siddhānta* diameters by two, we get 83,247.76 miles for Jupiter and 8,021.3 miles for Venus. These figures differ from the corresponding modern values by -4.15% and 6.10%. One can argue that the *Sūrya-siddhānta* diameters for Jupiter and Venus were actually the radii for these planets, and somehow they were accepted as diameters by mistake. If this mistake is corrected, then all of the *Sūrya-siddhānta* diameters for Mars, Saturn, Mercury, Jupiter, and Venus fall within 10% of the corresponding modern diameters.

Of course, it could be argued that this is all just number jugglery, and by juggling numbers one can make one thing correlate with just about anything else. But let us review the steps we have taken thus far. The calculations using formula (1) and the orbital circumferences in Table 2 were determined by the text of the *Sūrya-siddhānta*. The factor of 5 miles/*yojana* used to convert these calculated results into miles is reasonable and has been discussed by other authors.[13] There is no scope for juggling numbers here.

The only place where I have proposed an adjustment of the numbers is the multiplication of the *Sūrya-siddhānta* diameters of Jupiter and Venus by two. This suffices to bring these diameters into uniform agreement with modern data. To do the same thing with the Ptolemaic diameters would require a great deal more number jugglery than this, as the reader can see by examining Table 3. So if the *Sūrya-siddhānta* numbers can be so easily brought into line with modern data, then maybe this is because they have a genuine relationship with this data.

One possible explanation for this is that *śloka* 7.13 may have been written recently on the basis of modern planetary data and falsely interpolated into the text. This is ruled out, however, by the fact that there exists a manuscript of the *Sūrya-siddhānta* that is believed to have been written in the year A.D. 1431. This manuscript includes a commentary by Parameśvara, who died in A.D. 1450, and thus it definitely dates back to the 15th century. *Śloka* 7.13 is present in this manuscript, and it says the same thing in Sanskrit as the Burgess translation quoted above. The commentary explains the *śloka* point by point, and thus it confirms that the *śloka* was present in the manuscript in the same form in which it appears today.

In modern astronomy, the diameters of the planets are computed on the basis of accurate knowledge of their angular diameters and their distances from the earth. Both the angular diameters and the distances are determined by means of telescopic observations.

In 15th-century Europe the prevailing ideas concerning the sizes of the planets came from medieval Islamic astronomers who were following the teachings of Ptolemy. The first telescopic observations of planets were made by Galileo in 1609–10.[14] As late as 1631 Pierre Gassendi of Paris was shocked when his telescopic observation of a transit of Mercury across the sun revealed that its angular diameter was much smaller than he had believed possible.[15] It is clear that the information on planetary diameters in the *Sūrya-siddhānta* antedates the rise of modern astronomy.

If we hypothesize that *śloka* 7.13 incorporates knowledge of the actual diameters of the planets, then one natural question is this: If one started with the modern diameters of the planets and the *Sūrya-siddhānta* orbital circumferences, could one arrive at the rule given in this *śloka*? We can answer this question by solving equation (1) for DM(planet) and using the modern planetary diameter converted to *yojanas* in place of D(planet). We also use the radius in place of the diameter for Jupiter and Venus. The resulting values are listed in the leftmost column of Table 4.

The idea behind the rule in *śloka* 7.13 is to arrange the planets so that the computed DM values are in increasing order and then approximate them by a simple arithmetic progression. We can see from Table 4 that the order of the planets used in this rule does put the computed DM values in increasing order. One can approximate them by an arithmetic progression of the form ai+b either by trial and error or by using an optimization method such as least squares. I did this by least squares and got a=6.466 and b=32.584. This arithmetic progression is listed in the middle column of Table 4.

One could arrive at the rule in *śloka* 7.13 by observing that 32.854 is about 30, 45.515 is about 45, and 58.447 is about 60. Or one could compute the angular diameters listed in the rightmost column of Table 4 by dividing the numbers in the arithmetic progression by 15. It is plausible that someone looking for a simple rule might round off these angular diameters to the *Sūrya-siddhānta* series of 2, 2.5, 3, 3.5, 4.

Thus it is possible to derive the *śloka* 7.13 rule from modern values for the diameters of the planets. But how does it come about that this rule enables us to compute realistic approximations of these diameters? After the enormous simplification represented by the arithmetic progression 7.5i+30, it may seem strange that this rule could encode significant information about the true diameters of the planets. However, this information is contained in the order of the

Planet	Modern projection	Least sq. fit	Angular diameter
Mars	33.335	32.584	2.172
Saturn	36.545	39.050	2.603
Mercury	46.377	45.515	3.034
Jupiter	54.772	51.981	3.465
Venus	56.549	58.447	3.896

Table 4. In the leftmost column, modern planetary diameters are projected to the orbit of the moon, assuming the planetary orbits given in *Sūrya-siddhānta*. The projected diameters are expressed in *yojanas* (and radii are used in place of diameters for Jupiter and Venus). In the middle column, these projected diameters are fit to an arithmetic progression using least squares. The angular diameters in the rightmost column are obtained by dividing the figures in the middle column by 15 *yojanas* per minute of arc.

planets chosen in the *śloka* and the orbital circumferences of the planets.

Let us briefly consider these orbital circumferences. It turns out that these numbers are related to the geocentric periods of the planets. The geocentric period of a planet is the time that it takes for the mean position of the planet to make a complete circuit through the Zodiac and return to its starting place. As I pointed out before, the mean position for Mercury and Venus is the same as the position of the sun, and thus the sun, Mercury, and Venus all have geocentric periods of one earth year. For Mars, Jupiter, and Saturn, the mean position corresponds to the average motion of the planet in its heliocentric orbit, and thus the geocentric periods of these planets are the same as their years.

Table 2 lists the modern geocentric periods of the planets and their orbital circumferences according to the *Sūrya-siddhānta*. For each planet, the orbital circumference is a constant multiple of the period. This comes about because of the following rule in the *Sūrya-siddhānta* for computing the orbital circumferences:

> 12.81. If the stated number of revolutions of the moon in an Aeon (*kalpa*) be multiplied by the moon's orbit, the result is to be known as the orbit of the ether: so far do the sun's rays penetrate.
>
> 12.82. If this be divided by the number of revolutions of any planet in an Aeon (*kalpa*), the result will be the orbit of that planet: divide this by the number of terrestrial days, and the result is the daily eastward motion of them all.[16]

The idea behind this rule is that all of the planets move at the same mean rate in their geocentric orbits. Since the *Sūrya-siddhānta* gives accurate figures for the revolutions of each planet in a *kalpa,* this rule results in orbital circumferences proportional to the modern geocentric periods of the planets. (According to modern astronomy, the period squared should be proportional to the orbital circumference cubed.)

If we take this into account, then we can conclude that the *Sūrya-siddhānta* is saying the following about the diameters of the planets:

$$D(\text{planet}) = 7.5\,k\,P(\text{planet})/P(\text{moon}) \qquad (2)$$

where D(planet) is the planetary diameter in *yojanas*, P(planet) is the planet's geocentric period, P(moon) is the period of the moon, and $k=4$, 5, 6, 7, and 8 for Mars, Saturn, Mercury, Jupiter, and Venus.

This formula gives the same results as formula (1). If we use 14 and 16 instead of 7 and 8 for Jupiter and Venus (in effect doubling their diameters), then the root-mean-square error of the five computed diameters in relation to modern data is 5.8%.[17] We can regard this as an empiric fact about planetary diameters and periods. It is this fact which may have been used by the author of the *Sūrya-siddhānta* or his predecessors to derive the rule in *śloka* 7.13.

In summary, *ślokas* 7.13, 12.81–82, and 12.85–90 of the *Sūrya-siddhānta* contain information regarding the true diameters of the five planets Mercury, Venus, Mars, Jupiter, and Saturn. This information enables us to compute the diameters of three of these planets with errors of 10% or less. If the computed figures for Jupiter and Venus are interpreted as their radii rather than their diameters, then these radii are in error by about 4% and 6%, respectively. I hypothesize that this may not be due to mere coincidence. Rather, it may indicate that accurate knowledge of planetary diameters was possessed by ancient astronomers and used in the composition either of the *Sūrya-siddhānta* or of some earlier astronomical text on which it was based.

Appendix

In this appendix we set up a simple probabilistic model to evaluate whether or not the correlation between modern and *Sūrya-siddhānta* diameters shown in Table 3 could have arisen

Planet	*Sūrya-sidd.*	Ptolemy
Mars	3.0	20.2
Saturn	.7	19.7
Mercury	.9	39.6
Jupiter	19.4	23.3
Venus	17.1	28.5
Total	41.1	131.3

Table A1. Error angles in degrees for the planetary diameters listed in Table 3. The leftmost column compares *Sūrya-siddhānta* diameters with modern ones, and the rightmost column compares Ptolemaic diameters with modern ones.

by chance. Suppose that we randomly choose 5 numbers between 0 and B, where B is some fixed positive number. Call these numbers X(1), . . . ,X(5) and let them represent the *Sūrya-siddhānta* diameters of Mars, Saturn, Mercury, Jupiter, and Venus. Suppose we also randomly choose 5 numbers Y(1), . . . ,Y(5) between 0 and B to represent the modern values for these diameters. What is the probability that the X's will agree with the Y's as well as do the *Sūrya-siddhānta* and modern diameters listed in Table 3 for these five planets?

For each planet, draw a vector from the origin through the point (X,Y) representing the two diameter estimates for that planet. The angle A between this vector and the line y=x will be 0 if X=Y, and it can range from 0 to 45 degrees. We can call this the error angle for X and Y. If numbers X and Y are chosen at random between 0 and B, then A should fall between 0 and 45 degrees according to a certain probability distribution (proportional to $dA/(\cos(45\text{-}A))^2$), which turns out to be independent of the value chosen for B. Given this distribution, we can show that the sum of the 5 A's should have a theoretical mean value of 99.038 degrees and a theoretical standard deviation of 29.339 degrees.

The above table lists the values of the A's in degrees, using the modern and *Sūrya-siddhānta* diameters given in Table 3. This is also done using the Ptolemaic and modern values. It turns out that the total of the error angles for the *Sūrya-siddhānta* diameters is 41.1 degrees, which is about 1.98 standard deviations below the theoretical mean. This would be regarded as a statistically significant deviation from chance expectation, given the null hypothesis that the various diameters are chosen at random. In contrast, the total for the Ptolemaic diameters is 131.3 degrees, which is about 1.10 standard deviations above the mean. This is not very significant statistically. If anything, it shows a higher level of error than would be expected by chance.

Notes

1. Burgess, p. 195.
2. Swerdlow, Table 4.1 after p. 167.
3. Swerdlow, pp. 166, 168, 172.
4. Burgess, p. 196.
5. Motz and Duveen.
6. Burgess, p. 294.
7. Sarma, p. 84.
8. Parameśvara used a different measure for the *yojana*; his earth diameter of 1,050 *yojanas* yields about 7.6 miles/*yojana*. See Sarma, pp. 82–83.
9. These calculations were first reported in Thompson, Richard, *Vedic Cosmography and Astronomy*, Los Angeles: Bhaktivedanta Book Trust, 1989.

10. Swerdlow, Table 4.2 after p. 170.
11. The diameter of the moon's orbit in *Sūrya-siddhānta* is about 8.3% too large (at 5 miles/*yojana*), and its angular diameter of 32 minutes (=480/15) is about 2.6% too large.
12. The 6,500 *yojana* diameter of the sun and its orbital circumference of 4,331,500 *yojanas* imply that the sun's angular diameter is about 32.4 minutes.
13. For example, Sarma, p. 83, says that the *Sūrya-siddhānta* uses 5 miles/*yojana*, and Burgess, p. 44, gives a more exact figure of 4.94 miles/*yojana*. I have used least squares to find the number of miles/*yojana* that gives the best fit between the calculated and modern diameters of the moon, Mercury, Venus, Mars, Jupiter, and Saturn (with the calculated diameters of Jupiter and Venus doubled). This turns out to be 5.063 miles/*yojana*.
14. Drake, p. 154.
15. Van Helden, p. 5.
16. Burgess, pp. 293–94.
17. If we use 6.5 rather than 7.5 and let k = 5, 6, 7, 16, 18, then the root-mean-square error goes down to 4.3% This is close to optimal.

Bibliography

Burgess, Ebenezer, trans., *The Sūrya Siddhānta*, Gangooly, P., ed., Delhi: Motilal Banarsidass, 1989.

Drake, Stillman, Galileo's First Telescopic Observations," *Jour. for the History of Astronomy*, Vol. vii, 1976, pp. 153–168.

Motz, Lloyd and Duveen, Anita, *Essentials of Astronomy*, 2nd ed., New York: Colombia Univ. Press, 1977.

Sarma, K.V., trans., *The Goladīpikā by Parameśvara*, Madras: The Adyar Library and Research Center, 1956–57.

Swerdlow, Noel Mark, *Ptolemy's Theory of the Distances and Sizes of the Planets: A Study of the Scientific Foundations of Medieval Cosmology*, Yale Univ. Ph.D. thesis, 1968.

Van Helden, Albert, "The Importance of the Transit of Mercury of 1631," *Jour. for the History of Astronomy*, Vol. vii, 1976, pp. 1–10.